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Two-Link Flexible Manipulator Control Using Sliding Mode Control Based Linear Matrix Inequality

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Abstract. Two-link flexible manipulator is a manipulator robot which at least one of its arms is made of lightweight material and not rigid. Flexible robot manipulator has some advantages over the rigid robot manipulator, such as lighter, requires less power and costs, and to result greater payload. However, suitable control algorithm to maintain the two-link flexible robot manipulator in accurate positioning is very challenging. In this study, sliding mode control (SMC) was employed as robust control algorithm due to its insensitivity on the system parameter variations and the presence of disturbances when the system states are sliding on a sliding surface. SMC algorithm was combined with linear matrix inequality (LMI), which aims to reduce the effects of chattering coming from the oscillation of the state during sliding on the sliding surface. Stability of the control algorithm is guaranteed by Lyapunov function candidate. Based on simulation works, SMC based LMI resulted in better performance improvements despite the disturbances with significant chattering reduction. This was evident from the decline of the sum of squared tracking error (SSTE) and the sum of squared of control input (SSCI) indexes respectively 25.4% and 19.4%.

1. Introduction

Flexible robot manipulator is a robot manipulator, in which at least one of the links is made of a lightweight and flexible material. In general, flexible robot manipulator has several advantages than robot manipulator with rigid link. Flexible robot manipulator is lighter, require less power, low cost, and capable to result a larger payload [1].

Due to a lot of flexibility in its application, the two-link flexible manipulator is more preferable. However, to design a suitable control method for two-link flexible manipulator to maintain its accurate position is never easy. Two-link flexible manipulator has more complex problems than the manipulator with rigid link, because the flexible manipulator is a multi-input multi-output (MIMO) system. Other problems should be considered in the flexible manipulator are coupling problem in second link and vibration in between the two links [2]. Such that, an accurate model and efficient control method for the two-link flexible manipulator system must be developed.

A proper modelling is the main requirement for an optimum control method. In the previous studies, non-linear model was the model that frequently used in the system modelling. This type of model require a process that called as parameters identification and estimation. Implementation of this approach is based on the cutting of Taylor series until the first power degree around the certain point. Those modelling system has a drawback, in which the control systems cannot be synthesized to improve the performance of the system that is actually non-linear. Consequently, when the dynamics of the system is moving away from the operating point, then the performance of the controller can be reduced and the system can easily become unstable. It's due to availability of the control system only works properly in the nearest area of the operating point. The control of Proportional Integral Derivative (PID) is one of control methods that commonly used in industrial applications. However, PID controller becomes unstable when the system parameters changes and external disturbances exist. Moreover, it's not capable to drive states of the system to turn into its steady state with fast action [3].

In the class of nonlinear control, Variable Control Structure (VCS) via Sliding Mode Control (SMC) with specific characteristics is one of suitable answers for the problems of nonlinear systems. The main benefit of a control system using SMC is its insensitivity (robust) to parameter changes and disturbances. Moreover, SMC also be able to stabilize nonlinear system that can not be stabilized by using continuous state feedback control by using stability analysis [4]. The use of SMC method is expected to improve quality of the control algorithm with fast dynamic response, insensitive to parameter variations and disturbances, simple in control method, and can be implemented quite easy. Main factor affecting the performance of SMC are parameters of the sliding surface [5].

Regarding to the basic theory of SMC in [6], gain of the discontinuous control starts to result in discontinuous control action when the system states that are moving from somewhere reach a defined sliding surface and sliding on the surface toward their equilibrium point. However, the discontinuous control action displays a high frequency oscillation that is called as chattering. A higher gain will result a higher chattering amplitude. It is significantly undesirable and reduces the control performance [7]. In order to solve this drawback, several methods have been designed to reduce the chattering problem such as the use of a thin or varying boundary layer, reaching law method, low fast filtering, high-order SMC, adaptive robust control with disturbance observer, and any other methods.

In this study, SMC methods will be combined with linear matrix inequality (LMI) to obtain more robust control action and for chattering reduction [8-10]. The next section describes the flexible manipulator model in a nonlinear system platform. The proposed control technique is developed in section three. Then, the system and control parameters and set-up of the simulation are displayed in fourth section. The fifth section presents several discussions on results of the simulation works on the proposed technique in comparison with the conventional one. Conclusion of the study is appeared in the last section.

2. Model Formulation

Model of the two-link flexible manipulator expresses characteristics and behaviors of the system. The model was taken from the previous research [3], which created the system transfer function using system identification method from a set of input-output data taken from experiment. The complete mathematical model of the system is written as follow equation

$$G(s) = \frac{b_5 s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{1}$$

where,

$a_0 = 6.84,\ \ a_1 = 47.46,\ \ a_2 = 112.12,\ \ a_3 = 113.17,\ \ a_4 = 50.95,\ \ \ a_5 = 10.90$

$b_0 = 91.2,\ b_1 = 1080.3,\ b_2 = 2782.5.17,\ b_3 = 2571.1,\ b_4 = 840.9,\ \ b_5 = 84.1$

Mathematical model of the two-link flexible manipulator system in (1) can be changed into canonical form equation below

$$\dot{x} = (A_0 + \Delta A)x + B_0 u + d \tag{2}$$

Equation (2) can be rewritten by combining $\Delta A + d = d$

$$\dot{x} = A_0(x) + B_o u + d \tag{3}$$

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While, output of the system

$$y = Cx \tag{4}$$

where,

$$A_0 = \begin{bmatrix} -a_5 & -a_4 & -a_3 & -a_2 & -a_1 & -a_0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{5}$$

$$B_0 = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T \tag{6}$$

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T \tag{7}$$

$$Cx = [b_5 \; b_4 \; b_3 \; b_2 \, b_1 \; b_0] \, x \tag{8}$$

Substitute (5) to (7) into (3), model of the system can be represented in state-space canonical form:

$$\dot{x} = \begin{bmatrix} -a_5 & -a_4 & -a_3 & -a_2 & -a_1 & -a_0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [u + d] \tag{9}$$

$$\boldsymbol{y = Cx = [b_5 \; b_4 \; b_3 \; b_2 \, b_1 \; b_0] x} \tag{10}$$

In conversion of transfer function to state-space, substitute all parameters in (1) into (9) and (10), by ignorance disturbance.

3. Control Design

This control design is proposed to achieve a continuous control action in sliding mode control platform, such that, output of the system, x_i tracks the desired trajectories of the system states x_{id} as precise as possible in spite of parameters variation and the existence of disturbances [11].

3.1. Design of Sliding Mode Control

The proposed SMC is designed through several steps such as tracking error definition, sliding surface design, control law algorithm, stability analysis as follow:

3.1.1. Tracking Error. In order to design desired trajectories for the system states, the states errors of the system are defined as

$$e_i = x_i - x_{id} \qquad \text{where, } i = (1, 2, , .. , 6) \tag{11}$$

where,

$$x_1 = \theta_1, x_2 = \theta_2, x_3 = \alpha_1, x_4 = \dot{\theta}_1, \; x_5 = \dot{\theta}_2, x_6 = \dot{\alpha}_1 \tag{12}$$

Referring to the formula of Euler-Lagrange, model in (9) can be rewritten in state equation as below

$$\dot{x_1} = x_4$$
$$\dot{x_2} = x_5$$
$$\dot{x_3} = x_6$$
$$\dot{x_4} = -b_1 x_4 + b_2 x_3 + b_3 u$$

$$\dot{x_5} = -b_4x_5 - b_5x_3 + b_6u$$
$$\dot{x_6} = b_1x_4 - b_2x_3 - b_3u$$
$$\ddot{x}_1 = \dot{x}_4$$
$$\ddot{x}_2 = \dot{x}_5$$
$$\ddot{x}_3 = \dot{x}_6 \tag{13}$$

Such that tracking error for all states in the system as below

$$e = [e_1\ e_2\ e_3 e_4 e_5 e_6]^T \tag{14}$$

where,

$$e_1 = x_1 - x_{1d}$$
$$e_2 = x_2 - x_{2d} = \dot{x}_1 - \dot{x}_{1d}$$
$$e_3 = x_3 - x_{3d}$$
$$e_4 = x_4 - x_{4d} = \dot{x}_3 - \dot{x}_{3d}$$
$$e_5 = x_5 - x_{5d}$$
$$e_6 = x_6 - x_{6d} = \dot{x}_5 - \dot{x}_{5d} \tag{15}$$

3.1.2. Sliding Surface. A function of sliding surface needs to define to get attain the system states as defined in (12) track their defined desired trajectories at the same time [5], as follows

$$S = s_1 + \lambda_1 s_2 + \lambda_2 s_3 \tag{16}$$

where,

$$s_i = \dot{e}_i + c_i \int_0^t e_i(t)dt \tag{17}$$

$$s_i = \dot{e}_i + c_i e_i \tag{18}$$

where $\lambda_i (i = 1,2)$ and c_i $(i = 1,2,3)$ is a positive constant which is determined in accordance with the desired dynamics and the closed-loop system. *S* parameter is called as the sliding surface. The stability of the system relies on the constant.

Sliding surface is given $S = 0$ for the desired dynamic response of the system. If *S* is driven to zero, the desired dynamics is reached and the tracking error will converge to zero.

*3.1.3. Reaching Law.*In order to boost states of the system to reach the sliding surface as fast as possible, the reaching time equation is needed. The dynamic equation of constant reaching law is given as in [5].

$$\dot{\boldsymbol{S}} = -\boldsymbol{\rho}\, \boldsymbol{sign}\, (\boldsymbol{S}) \tag{19}$$

where, $\rho > 0$. Smaller value of ρ is suggested to reduce chattering effect.

3.1.4. Control Law. Control law will bring the system states trajectories towards the sliding surface which has been designed in advance and kept it in sliding condition. In general, the control input is m-vector *u(t)*. In order to determine the control law, the sliding surface (16) is defined in derivative form

$$\dot{\boldsymbol{S}} = \dot{\boldsymbol{s}}_1 + \boldsymbol{\lambda}_1 \dot{\boldsymbol{s}}_2 + \boldsymbol{\lambda}_2 \dot{\boldsymbol{s}}_3 \tag{20}$$

where,

$$\dot{\boldsymbol{s}}_1 = \ddot{\boldsymbol{e}}_1 + \boldsymbol{c}_1 \dot{\boldsymbol{e}}_1$$

$$\dot{s}_2 = \ddot{e}_2 + c_2\dot{e}_2$$

$$\dot{s}_3 = \ddot{e}_3 + c_3\dot{e}_3$$

By substituting Equation (11), (13), and (19) into (20), then control law of the system is obtained

$$\boldsymbol{u} = \left(-\frac{\boldsymbol{1}}{\boldsymbol{k}}\right)[x_6(\lambda_2 c_3) + x_5(-\lambda_1 b_4 + \lambda_1 c_2)\lambda_2 b_1 + x_4(-b_1 + \lambda_2 b_1 + c_1) + x_3(b_2 - \lambda_1 b_5 - \lambda_2 b_2) - \lambda_2 - (c_3 e_3) - e_1 + (\lambda_1 c_2 e_2)] - \rho\, sign\,(S) \tag{21}$$

where, $\boldsymbol{k} = \boldsymbol{b_3} + \boldsymbol{\lambda_1 b_6} - \boldsymbol{\lambda_2 b_3}$, and c_1, c_2, c_3 are given.

3.2. Stability Analysis

One of the main issues in control system design is how to guarantee stability of the control system. In this study, stability of the proposed control algorithm is assured by a Lyapunov function candidate.

$$V\,(S) = \frac{1}{2}S^2 \tag{22}$$

The Lyapunov stability criterion of the proposed control is defined as

$$\dot{V}(S) > 0 \tag{23}$$

Then, to reach the condition in (23), it follows that

$$\dot{V}(S) = \frac{1}{2}\frac{d}{dt}S^2 = S\dot{S} \ll -\eta - \rho\, sign\,(S) \tag{24}$$

where, $\eta > 0$.

Hence, ρ in Equation (19) can be determined by substituting Equations (16), (19) and (20) into (24). While, the control design was examined by using Matlab Simulink as in Figure 1 below

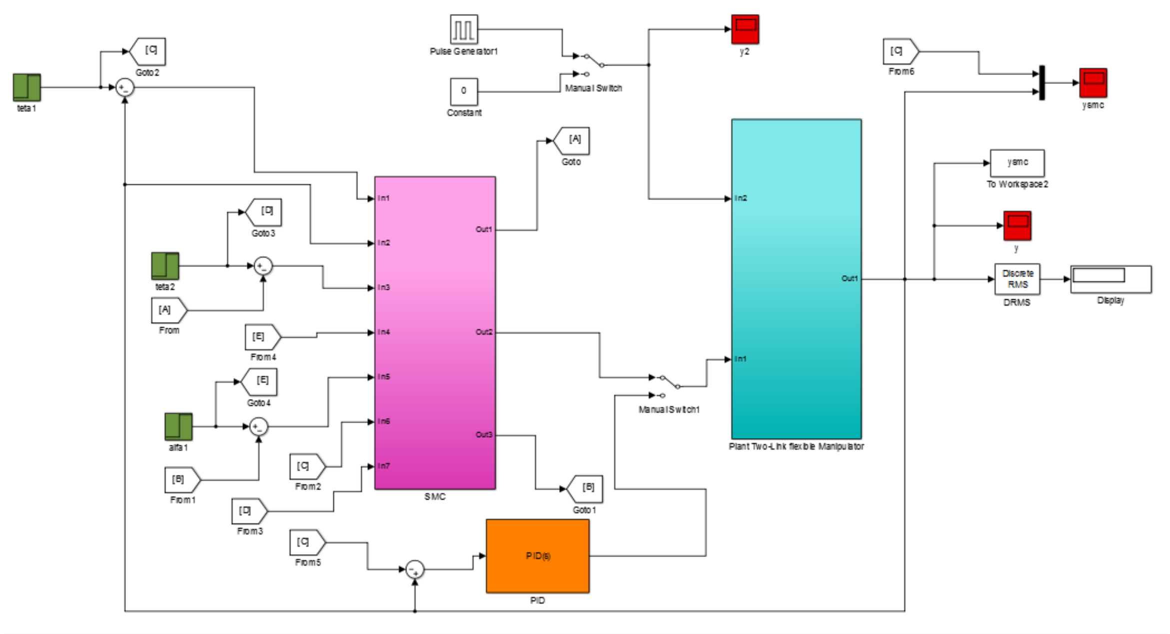

Figure 1. SMC design in Matlab Simulink

3.3. Design of Sliding Mode Control Based Linear Matrix Inequality

In this part, SMC based LMI is designed to complete the SMC design in Figure 1 above in order to decrease effect of chattering on control action of the SMC. The SMC based LMI designed is represented by block diagram in Figure 2.

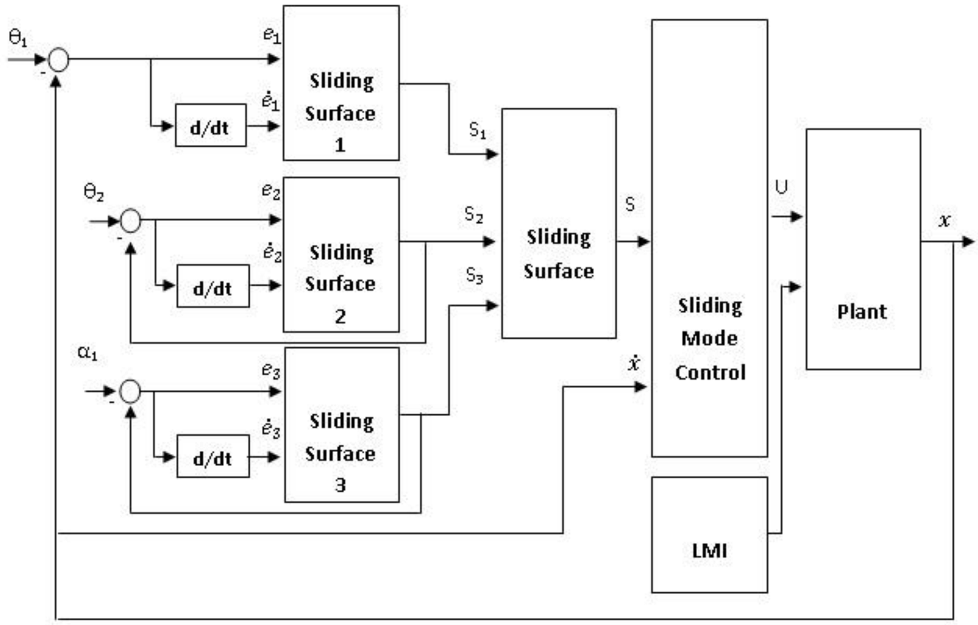

Figure 2. Block diagram of SMC based LMI

Then, the steps to design LMI are corresponded to flow chart in Figure 3 below

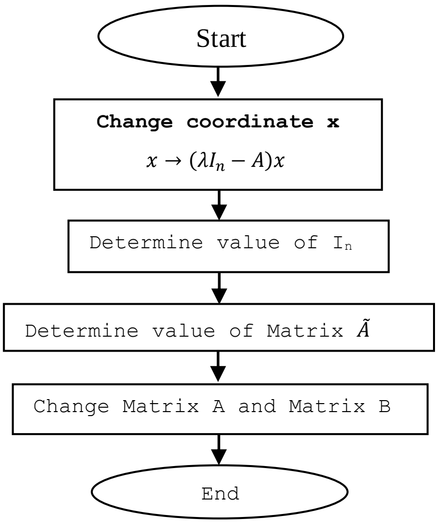

Figure 3. Flowchart of LMI Design

Before design the control algorithm, firstly, determine parameters of the control design. Control parameters are obtained from Equation (3), by assuming that A, B, C, D are constant matrices, which are parts of the state equation of the system model. Design of the optimized SMC with the LMI can be done by changing the initial coordinates of x.

$$x \to (\beta I_n - A)x \tag{25}$$

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When β is not located along the spectrum A and a non-singular transformation, a matrix structure will be replaced with new coordinates.

$$\tilde{A} = (\beta I_n - A)A(\beta I_n - A)^{-1} \tag{26}$$

$$\tilde{B} = (\beta I_n - A)B \tag{27}$$

β is a positive constant. It is set in 9,887, which is gotten from trial and error. While I_n is the 6 x 6 identity matrix. Having obtained the value of $\tilde{A}$ and the value of$\tilde{B}$, then both parameters were included in the state-space block in the MATLAB Simulink.

Such that, matrices $\tilde{A}$ and $\tilde{B}$ are obtained as below

$$\tilde{A} = \begin{bmatrix} -988.182 & -6809.556 & -17833.656 & -18956.899 & -8297.994 & -1215.942 \\ 177.862 & 951.253 & 2252.238 & 2294.929 & 984.645 & 142.827 \\ -20.892 & -49.948 & -113.169 & -112.117 & -47.456 & -6.836 \\ 1 & -9.988 & 1 & 0 & 0 & 0 \\ 0 & 1 & -9.988 & 1 & 0 & 0 \\ 0 & 0 & 1 & -9.988 & 1 & 0 \end{bmatrix} \tag{27}$$

$$\tilde{B} = \begin{bmatrix} 6.836 & 0 & 0 & 0 & 0 & 9.887 \end{bmatrix}^T \tag{28}$$

4. Simulation Setup

Simulation is done in evaluating performance of the controller algorithm by delicately tune its parameters. In this study, the initial SMC and the proposed SMC based LMI abilities are assessed in minimizing chattering effect on the control signal of the SMC when the system parameters changes and unknown disturbances exist.

How to reduce chattering in the control action of the SMC is the main task of this work. In examining the smoothness of control signal, a step reference signal is employed in a transient performance assessment. The reference signal will facilitate the investigation on the chattering effect when the system reaches its steady state condition. The simulation works employ Runge-Kutta solver with a sampling time of 0.001s. Sum of squared tracking error (SSTE) and sum of squared control input (SSCI) are used to measure index of the control action performance. The SSTE is employed to measure the system tracking accuracy, and the SSCI is used to detect the control activities when LMI added to the initial SMC design.

$$SSTE = \sum_{t=0}^{N} (x_{1d}(t) - (x_1(t))^2 \tag{29}$$

$$SSCI = \sum_{t=0}^{N} u(t)^2 \tag{30}$$

5. Results and Discussion

Detail of observations through step input signal assessment on the transient response of the proposed design of SMC relates to its consideration on chattering reduction when system parameters changes and disturbances exist are carried out in this section. Figure 4 to Figure 6 presents the output responses of the system, tracking error, and control signals of the initial SMC in comparison with SMC based LMI. It can be detected by comparing the first and the second version of the control design in tracking error and control action performance. The displayed figures are supported by a comparison table of SSTE and SSCI as present in Table 1.

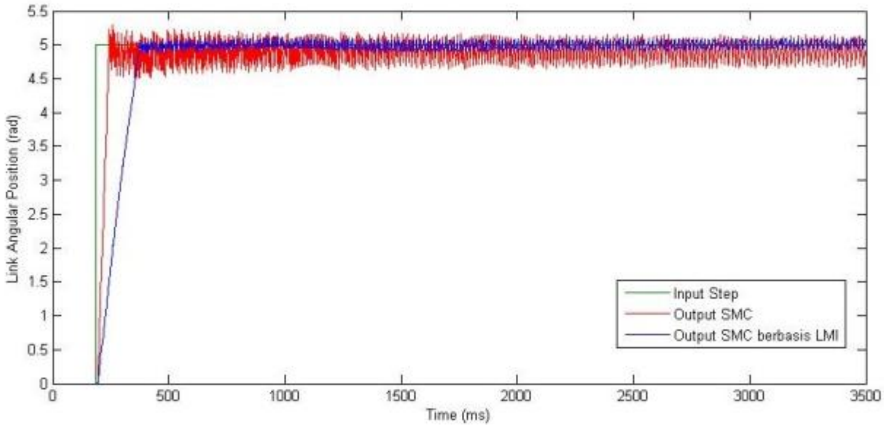

Figure 4. Output response of SMC and SMC based LMI

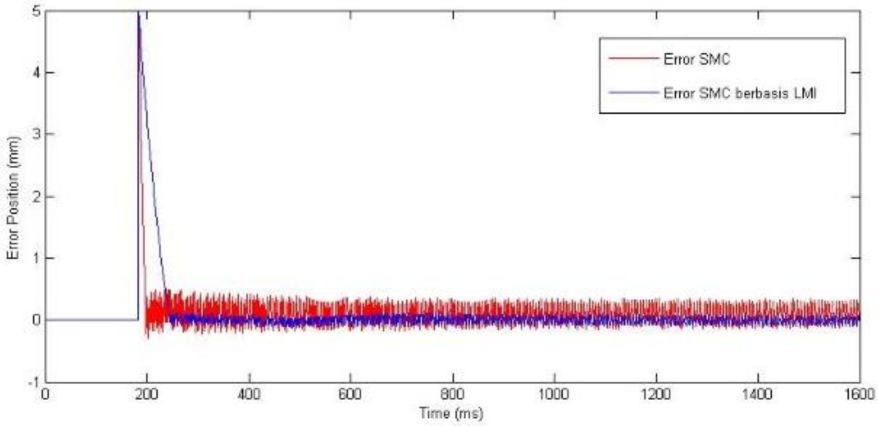

Figure 5. Tracking error of SMC and SMC based LMI

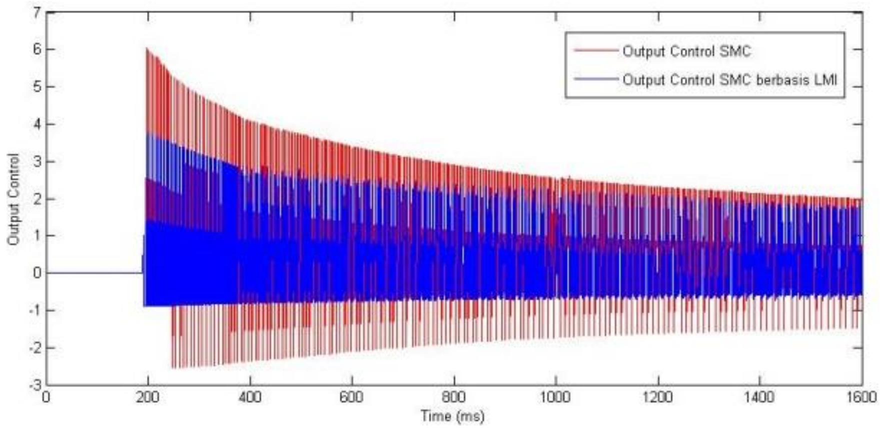

Figure 6. Control signals of SMC and SMC based LMI

As can be seen from Figure 4 to Figure 6, SMC based LMI was capable to minimize chattering compare to the conventional SMC. Transformation of the system model by using LMI guarantees the system become more robust and stable in the face of parameters changes and disturbances. Since the control gain of the SMC was calculated from the transformed model, it was possible to obtain a smaller control gain. In basic theory of SMC has already state that, a smaller control gain will produce smaller chattering in the control signal. This conclusion also was justified by comparison SSTE and SSCI indexes in Table 1, which display that SSTE and SSCI of SMC based LMI significantly smaller than SMC conventional.

Tabel 1. Control Performance Comparison

Controller/Index	SSTE	SSCI
SMC	0.106	0.237
SMC based LMI	0.079	0.191

6. Conclusion

Each Model of the two-link flexible manipulator has been successfully formulated from the existing transfer function form into state equations. This state equations model is required to develop SMC algorithm. The proposed SMC algorithm also has been developed successfully for the flexible manipulator system to be robust and insensitive on parameters variation and disturbances. Through step input signal test, it was seen that the development of SMC with LMI has increased capability of the SMC to be more robust and result smaller chattering in its control action. This was verified from the decline of the SSTE and SSCI indexes around 25.4% and 19.4%, respectively.

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